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Application of the Monte Carlo Scanning method to the allocation of anti-aircraft artillery

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ABSTRACT: The Monte Carlo Scanning Method was developed in the Decision Support Systems Subdivision of the Institute for Advanced Studies to explore the speed of modern computers. The strategy used to address optimization problems seeks the thorough random sampling of the domain and the assessment of the objective function. The Monte Carlo Scanning Method is applied to an anti-aircraft artillery allocation problem. The results obtained for the analyzed problem are identical to results of the literature which used Dynamic Programming. Simplicity, facility for implementation, and the ability to quickly find maximized solutions indicate that the Monte Carlo Scanning Method can be considered as a powerful computational simulation tool for military decision support systems.

KEYWORDS: Monte Carlo simulation. Allocation of anti-aircraft artillery. Operational analysis.

RESUMO: O Método Monte Carlo Scanning foi desenvolvido na Subdivisão de Sistemas de Apoio à Decisão, do Instituto de Estudos Avançados, visando explorar a velocidade dos computadores atuais. A estratégia usada para abordar problemas de otimização tem como princípio a amostragem aleatória exaustiva do domínio e a avaliação da função objetivo. O Método Monte Carlo Scanning é aplicado a um problema de alocação de artilharia antiaérea. Os resultados obtidos para o problema analisado são idênticos aos resultados da literatura, que utilizaram Programação Dinâmica. A simplicidade, a facilidade de implementação e a capacidade de encontrar rapidamente soluções maximizadas indicam que o Método Monte Carlo Scanning pode ser considerado como uma poderosa ferramenta de simulação computacional para sistemas de apoio à decisão militar.

PALAVRAS-CHAVE: Simulação Monte Carlo. Alocação de artilharia antiaérea. Análise operacional.

1. Introduction

Technological development in the military area has become complex, requiring decision support systems for operational analysis. These systems should help resolve project, effectiveness, cost, and allocation issues of defense or attack weapons. These issues usually involve maximizing or minimizing certain quantities, such as financial, material, and human resources, weapons, logistical support, etc.

The optimization process consists of finding a function of multiple variables, which represents the problem and is subject to constraints to be maximized or minimized. It uses deterministic methods such as linear programming, dynamic programming, nonlinear programming, etc. Another class of methods, called heuristic or stochastic methods, includes the Monte Carlo Simulation, Simulated Annealing (SA), genetic algorithms, stochastic neural networks, among others. The complexity of real problems usually requires using stochastic or heuristic methods [1].

Nicholas Metropolis, Stanisław Ulam, Enrico Fermi, John von Neumann, and Edward Teller are considered [2] the creators of the Monte Carlo Methods. Rand Corporation researchers supported by the U.S. Air Force have made several developments to the Monte Carlo Methods, which have been applied [3,4] in various areas of Science and Technology, including operational research.

The Subdivision of Decision Support Systems of the Institute for Advanced Studies has encouraged using heuristic methods to solve optimization problems. Several studies have therefore been conducted [5-7] using the SA Method.

Considering the low processing speed of computers in the past, the applications of Monte Carlo Methods were very limited. However, with the evolution of processor speed (Table 1) [8], these methods have become practically mandatory for some classes of problems. The fastest computer in Brazil, which is in the National Laboratory of Scientific Computing (LNCC), is 7000 times faster than the PC used in this article.

Tab. 1 - Selected Linpack Benchmarks

Name	Description	TOP500	GFLOPS
SunwayTaihuLight	Jiangsu, China	1	93,014.600
Tianhe-2	National University of Defense Technology, China	2	33,862.700
Titan	DOE/SC/ORNL	3	17,590.000
10692 x Intel Xeon E5-2695v2	LNCC, Brazil	265	456,800
2 x Intel Xeon E5 2687W v4	2016	-	1,078
Intel Core I5 3330 CPU 3,00 GHz	PC used in this article	-	64
Intel Core I7, 3.20 GHz, 4 cores	Standard PC in 2009 (64-bit)	-	33
Intel Pentium II, 450 MHz	Standard PC in 1999	-	0.4
Intel 386 DX, 33 MHz	Standard PC in 1989	-	0.008

The Monte Carlo Scanning Method (MCS) was developed to show that the speed of modern computers allows finding optimal solutions just by sweeping the domain and inspecting the image of the objective function. The MCS Method uses the basic principle of Monte Carlo, according to which random quantities evenly distributed between [0,1] are used for the simulation of events that virtually obey any distribution law. Its main advantages over other methods are simplicity, robustness, and speed of computational implementation.

This method was developed both because of the large increase in the speed of computers over time and of the difficulty of tuning the convergence parameters of the SA Method, considered a major obstacle for applications in computer programs used daily. In this line, researchers conducted optimization studies of continuous functions [9] and of a discrete function, characteristic of the issue of allocation of air defense weapons [10-12], to compare the results obtained with those of other methods, verifying the methodology.

One of the many problems of weapon allocation that may require support systems for military decision-making is the allocation of Anti-Aircraft Artillery (AAA) for the defense of different types of installations [1]. This issue has been previously resolved [1] using the Dynamic Programming Method (DP) [13]. In this study, this same problem was analyzed using the MCS Method.

Section 2 describes the AAA allocation issue and Section 3 briefly addresses the methodologies [1,9] used for the problem. Section 4 shows the results achieved and Section 5 presents final comments and recommendations for future studies.

2. Allocation of anti-aircraft artillery

The AAA allocation issue [1] considers that a planner should distribute *NAAA* anti-aircraft batteries for the protection of *NSites* installations. The survival of each installation is described for each level of protection, i.e., when unprotected, protected by a battery, by two batteries, and so on.

In the case analyzed, *NAAA* equals six and *NSites* equals four. **Table 2** shows the Installation Survival Probabilities (ISP). **Figure 1** shows data from **Table 2** on a graphic, showing that Installation 4 has higher survival probabilities than Installation 3, which, in turn, has higher probabilities than Installations 1 and 2. Initially, for small numbers of allocated anti-aircraft batteries, Installation 1 has lower survival probabilities than Installation 2; however, when the number of batteries increases, this behavior is reversed.

Tab. 2 – Survival of installations (%)

Anti-aircraft batteries	Installation			
	1	2	3	4
0	30	40	40	40
1	40	50	50	50
2	50	60	60	60
3	60	60	70	80
4	70	60	80	90
5	70	60	80	90
6	70	60	80	90

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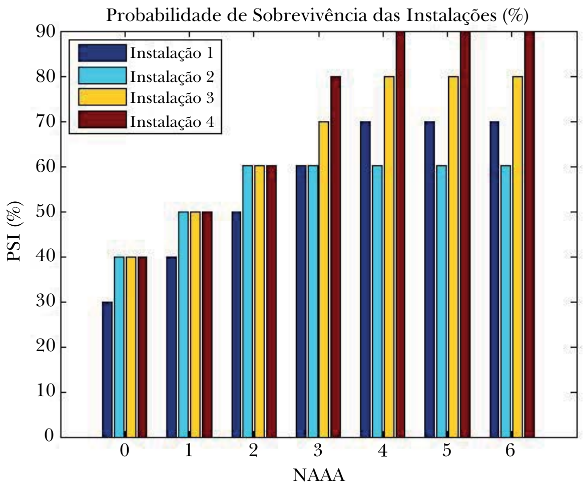

Fig. 1 – Survival probability of the installations (%).

3. Methodologies

The DP Method has been used extensively in military operational research for its ability to address several real problems [1]. However, this method requires great experience on extensive and complex algorithm construction, especially for problems with several variables that produce high-order matrices.

Solving the AAA allocation issue with the DP Method shows the method's main mechanisms, which are the use of recursive formulas and the maintenance of maximum values of intermediate solutions to find the maximum final solutions [1].

The MCS Method uses the basic principle of Monte Carlo and seeks to explore the speed of modern computers. It can be efficiently used by correct domain sampling and the establishment of an appropriate objective function.

In the construction of samples with *NAAA* anti-aircraft batteries distributed in *NSites* installations, a generating function was used for the sampling of *NSites* random integers *Xd* in the range [0,*NAAA*] whose sum is also *NAAA*. However, this procedure introduces a bias that compromises the uniformity of sampling. The sampling should thus be randomized to correct this bias.

The effectiveness of the total protection of the installations is achieved by maximizing the total survival value of the *NSites* installations, sweeping the various possible combinations, expressed by the Objective Function (*OF*)

$$FO = \sum_{i=1}^{NSites} PSI(Xd(i) = 1, i), \qquad (1)$$

subject to restriction

$$\sum_{i=1}^{NSites} Xd(i) = NAAA, \qquad (2)$$

where *Xd(i)* represents the sampled number of anti-aircraft batteries for each Installation *i*.

In short, the MCS Method is applied to obtain a valid sample and estimate the objective function, which in this case is the sum of the survival probabilities (%) of each installation. After several samples are processed, repeated samples are eliminated and those with the highest objective function constitute the group of solutions of maximum effectiveness. **Table 3** shows, step by step, an algorithm of the MCS Method for AAA allocation.

Tab. 3 – MCS Method algorithm for AAA allocation optimization.

Step	Action to be performed
1	Show *NSites* integers $Xd \in [0, NAAA]$ whose sum equals *NAAA*.
2	Ensure that the *Xd* is evenly distributed by randomization of the *Xd* vector.
3	Estimate *OF*.
4	Repeat Steps to 3 a sufficient number of times to ensure a good statistic. In the analyzed situation, 1000 samples were used.
5	Delete repeated *Xd* values.
6	Sort AAA allocation plans according to decreasing *OF* values.
7	Choose the plans with the highest *OF* values.

4. Results

The results achieved with the MCS Method, implemented in the MCS-AAA computer program using the Matlab environment [14], are equal to the results published in the literature [1]. **Table 4** shows the fifteen independent solutions, obtained considering 1000 samples.

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Tab. 4 – Optimized allocation plans for AAA Batteries

Plan	Allocation/installation				Survival (%)				Objective function
	1	2	3	4	1	2	3	4	
1	0	0	2	4	30	40	60	90	220
2	0	0	3	3	30	40	70	80	220
3	0	1	1	4	30	50	50	90	220
4	0	1	2	3	30	50	60	80	220
5	0	2	0	4	30	60	40	90	220
6	0	2	1	3	30	60	50	80	220
7	1	0	1	4	40	40	50	90	220
8	1	0	2	3	40	40	60	80	220
9	1	1	0	4	40	50	40	90	220
10	1	1	1	3	40	50	50	80	220
11	1	2	0	3	40	60	40	80	220
12	2	0	0	4	50	40	40	90	220
13	2	0	1	3	50	40	50	80	220
14	2	1	0	3	50	50	40	80	220
15	3	0	0	3	60	40	40	80	220

Figure 2 shows the distribution relative to **Table 4**. Most AAA batteries were allocated in Installation 4.

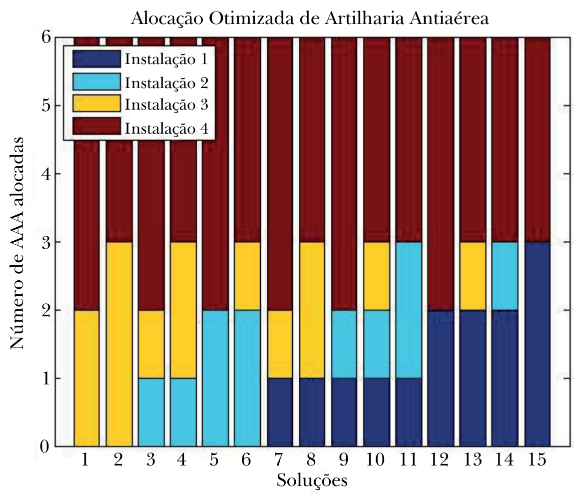

Fig. 2 – Optimized plans with the number of AAA batteries allocated in each installation.

We also analyzed a situation, in which planning requires that all installations have a survival greater than 50%. **Table 5** shows the four maximized solutions for this case. These results are also identical to those published in the literature [1].

Tab. 5 – Optimized plans for installation survival values greater than or equal to 50%

Plan	Allocation/installation				Survival (%)				Objective Function
	1	2	3	4	1	2	3	4	
1	2	1	1	2	50	50	50	60	210
2	2	1	2	1	50	50	60	50	210
3	2	2	1	1	50	60	50	50	210
4	3	1	1	1	60	50	50	50	210

Figure 3 shows the distribution relative to **Table 4**, indicating that the solutions have at least two AAA batteries allocated in Installation 1, as expected according to **Figure 1**.

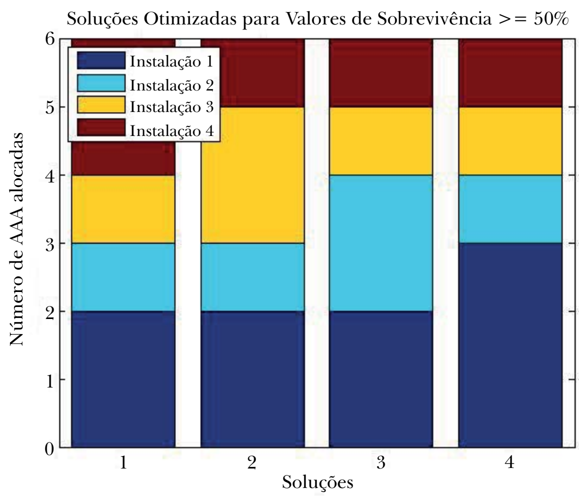

Fig. 3 – Optimized plans with the number of AAA batteries allocated considering survival values greater than or equal to 50%.

One way to find a sufficient number of samples is to observe the behavior of the solutions obtained. For a heuristic method, if the number of solutions remains constant after increasing the number of samples, then all solutions may have been found. Other methods can ensure the quality of results, but in this article, we observed only the behavior of the solution.

5. Final remarks

For the proposed AAA allocation issue, the MCS Method – with only 1000 samples, which represents a small computing time – presented solutions equivalent to those obtained in the literature with the

DP Method for the issue of AAA battery allocation. Therefore, the simplicity, which implies great facility of implementation, and the computational effectiveness of the MCS Method are very attractive to software such as the AEROGRAF Platform [15]. The next activity, in this line of research, will be to implement the MCS-AAA software as a plug-in of this platform. However, for interested parties, the software can be made available upon request.

Our next study will seek to introduce an optimization procedure in the MCS Method based on a refinement of the solutions found. Finally, a computer program will be developed to solve the allocation issue of airstrike weapons using the MCS Method.

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