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Physical approach for resistive force in the modeling of internal ballistics by lumped parameters method

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ABSTRACT: *This study proposes a formulation based on physical parameters to represent the resistive force in a modeling of internal ballistics by lumped parameters method. The parameters of the model were adjusted by the solution of an inverse problem performed with a view to approaching a reference behavior to the pressure in the chamber according to the position of the projectile. We considered the results adequate and, considering the reference behavior to the M80 cartridge in a 24-inch barrel, we found that the proposed formulation is more accurate than the one that gave it its origin.*

KEYWORDS: Internal ballistics, Modeling, Lumped Parameters, Resistive Force

RESUMO: O presente trabalho trata sobre a proposição de uma formulação baseada em parâmetros físicos para representar a força resistente em uma modelagem da balística interna por uma abordagem via parâmetros concentrados. Os parâmetros do modelo foram ajustados pela solução de um problema inverso realizado com vistas à aproximação de um comportamento de referência para a pressão na câmara em função da posição do projétil. Os resultados foram considerados adequados e, tendo em vista o comportamento de referência para o cartucho M80 em um cano de 24 polegadas, constatou-se que a formulação proposta é mais exata que a que lhe dera origem.

PALAVRAS-CHAVE: Balística interna, Modelagem, Parâmetros Concentrados, Força de Resistência.

1. Introduction

Internal ballistics relates the loading characteristics, e.g., gunpowder composition and grain geometry, with the features of the projectile and weapon, considering the achievement of a certain muzzle velocity.

Figure 1 shows the basic appearance of a weapon and its operation. The propellant in the case produces gases, represented by force F_g, which propels the projectile forward, accelerating it to the muzzle of the gun. On the other hand, the barrel of the weapon provides a resistance to the projectile displacement, represented by force F_{res}.

There are several ways to address the issue of internal ballistics. The simplest way is using lumped parameters, in which space and vector quantities are represented by scalar variables. This type of approach provides a simpler solution thanks to the small number of degrees of freedom when compared with those existing in the case of the solution of a fluid-structure interaction problem with a two-phase flow by some numerical method [1][2].

However, the modeling for internal ballistics still challenges, once there are parameters with little-studied behaviors. We can mention, for example, lost energy and resistive force.

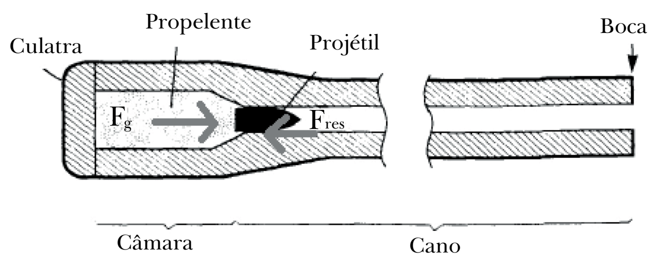

Fig. 1 – A weapon with its components and the forces applied to the projectile. Source: adapted from [3].

1.1 Bibliographic review

One of the main works in the area of internal ballistics is a book of Hunt [1]. His book was one of the pioneers in the area and still serves as the basis for some works. Hunt presents a semi-empirical model for lost energy and suggests us to represent the resistive force as an additional loss in the energy balance. He suggests that this loss is a percentage of kinetic energy, from 4% to 5%.

Another important work in the area, which serves as the basis for one of the main software in the ballistics field—PRODAS—is that of Baer-Frankles [2]. They use Hunt's approach; however, they suggest the incorporation of the coefficients K_x and K_v in the burning equation, implying that the position and velocity of the projectile are relevant factors for the propellant burning behavior. The authors also included the resistive force, obtained experimentally, as an input datum.

To implement the model proposed by [2] to the 7.62 × 51 mm (M80) cartridge, [3] uses the resistive force profile available in the PRODAS database. Meanwhile, by the solution of an inverse problem, [4] adjusts the burning parameters in [3]'s model to reproduce the curves present in the PRODAS database for the M80 cartridge.

1.2 Objectives

Within this context, this study aims to expand a previous study [4] by modeling the resistance to the advance of the projectile based on physically representative parameters, and improving both the treatment of the lost energy and the understanding about the ballistic behavior of the ammunition due to changes in its loading, environmental conditions of operation, or the barrel that fires them.

The justificative of this study relies on its potential use in fields such as internal ballistics, dynamics of automatic or semi-automatic armament mobile components, and dynamics of heavy weapons recoil damping systems.

2. Definitions

The inner surface of the gun tube or barrel through where the projectile moves is called bore. A gun may have a smooth bore or a rifled bore. Smooth-bore weapons present an uniform bore surface, whose diameter is called caliber, while the rifled-bore ones present riflings, and consequently a diameter relative to the lands of the riflings, the caliber, and another relative to the grooves, the bottom of the riflings.

The modeling addressed in this study applies both to light weapons (caliber less than 0.6 inch) and heavy weapons. Some say light weapons have a barrel, whereas heavy weapons have a tube. As the numerical case of our study is a light weapon ammunition, we are always going to use the term barrel, to keep the text of the article uniform, although we understand the formulation also applies to tubes.

Light weapons munitions, with rare exceptions, are stored in cartridges and embedded. It means that they are composed of projectile, propellant, case, and primer, and that this set is delivered already assembled, ready for shooting. Dislodging force is the force required to separate the projectile from the neck of the case.

The mesh refers to the riflings fitting the projectile, which has a diameter greater than the one of the bottom of the rifling. Thus, in the forcing cone (**Figure 2**), while the lands cause plastic deformation, rifling the projectile, it is compressed by the bottom of the riflings, obturating, which prevents the propellant gases from advancing beyond the projectile.

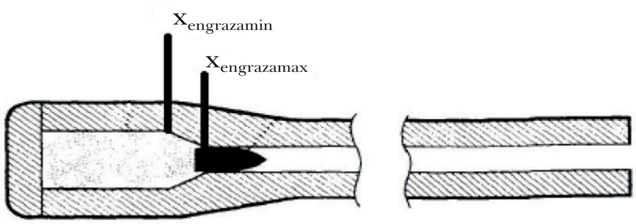

Fig. 2 – Forcing cone bounded by $x_{minmesh}$ and $x_{maxmesh}$. Source: adapted from [3].

PRODAS is a ballistics software that has an extensive database of ammunition ballistic behaviors. Therefore, the internal ballistics curves of the projectile used in this study can be obtained from it to function as a reference to the adjustment of parameters and validation of models.

3. Modeling

This section discusses the equation required to describe the phenomenon of the projectile displacement along the barrel as the propellant burns. We divided the equation into burning law, lost energy, resistive force, and projectile dynamics equations.

3.1 Burning law

For internal ballistics, we considered, a priori, Piobert's law. We considered that all surfaces of the propellant grains are ignited at the same time and that the entire surface is consumed at the same rate at each instant, and in form of parallel layers [5]. We present the equations below. Equation 1 is the propellant burning equation; in this case, **P** is the pressure of the gases inside the chamber, **x** is the position of the projectile in the barrel, $\mathbf{V_{el}}$ is the velocity of the projectile. Equation 2 finds the volumetric fraction of burned propellant, called **z**, from **f**, which is the remaining fraction of ballistic length of the propellant. Equation 3 is the expression of the pressure of a polytropic gas expansion and **ω** is the angular velocity of the projectile, **A** is the transverse area of the inside of the barrel [5].

$$\frac{df}{dt} = \frac{B}{D}P^a + K_x x + K_v V_{el} \tag{1}$$

$$z = (1-f)(1+kf) \tag{2}$$

$$P = \frac{CzF - (y-1)(0.5MV_{el}^2 + 0.5M\omega^2 + E_{lost})}{V_o + Ax + Cz(\frac{1}{d_{ensi}} - c_{ovol})} \tag{3}$$

3.2 Lost energy

For lost energy, $\mathbf{E_{lost}}$, [2] presents a semi-empirical modeling that relates the lost energy in terms of weapon parameters. $\mathbf{V_{elmuzzle}}$ is the velocity of the muzzle of the weapon. In this equation, we have to use inches for the caliber and pounds for the initial mass of the propellant. See that the velocity of the muzzle of the weapon must be known.

$$E_{lost} = \frac{[0,38Ca^{1,5}L(T_o - T_S)]}{1+0,6Ca^{2,175}/C^{0,8375}}\frac{V_{el}^2}{V_{elmuzzle}^2} \tag{4}$$

3.3 Resistive force

We considered four components in the formulation of the resistive force:

- the dislodging force, presented in Equation 5, is the component due to the dislodgment of the projectile in the case, and it acts while the projectile does not leave its initial position;

$$F_{dislodging}(x) = \begin{cases} F_{dislod}, & x = 0 \\ 0, & x > 0 \end{cases} \tag{5}$$

- the friction force, due to obturation [6], Equation 6, is the component due to the sliding of the surface of the projectile on the bottom of the bore rifling. It acts only while the projectile moves in the barrel bore.

$$F_{obturation} = \begin{cases} 0, x < x_{minmesh} \\ F_{obtu}, x \geq x_{minmesh} \end{cases} \tag{6}$$

- the mesh force, shown on Equation 7, is the component that acts on the projectile when it passes through the forcing cone of the weapon. This cone is delimited by the parameters $\mathbf{x_{minmesh}}$ and $\mathbf{x_{maxmesh}}$, presented in **Figure 2**.

$$F_{mesh} = \begin{cases} F_{mesh}, x_{minmesh} < x < x_{maxmesh} \\ 0, \textit{if not} \end{cases} \tag{7}$$

- the force due to the rifling, the axial component of the forces the riflings exert on the projectile, as [6] shows, is estimated according to Equation 8. In it, α is the rifling angle of the barrel and μ is the friction coefficient between the projectile and the barrel. In the proposed model, we disregarded the friction caused by the projectile distension with the barrel grooves, since, according to [7], this effort is negligible for light weapons munitions.

$$F_{rifling} = \frac{I_o}{\left({}^{C_a}/_{2}\right)^2}(sen\alpha + \mu cos\alpha)\ldots$$

$$\ldots\left(\frac{\frac{dV_{el}}{dt}tg\alpha + V_{el}^2\frac{dtg\alpha}{dx}}{\cos\alpha - \mu sen\alpha}\right) \quad (8)$$

Thus, we can estimate the resistive force by its components, as Equation 10.

$$F_{res} = F_{dislodging} + F_{obturation} + F_{mesh} + F_{rifling} \quad (9)$$

3.4 Projectile dynamics

To complete the set of equations and completely define the system, we must apply Newton's First Law to the projectile, as presented in Eq. 10.

$$F = PA - F_{res} = M\frac{d^2x}{dt^2} \quad (10)$$

4. Methods

4.1 Direct problem

By the lumped parameters method proposed by [4], we elaborated the following arrangement to solve the initial value problem (IVP), composed of the order reduction of Equation 10 with Equation 1.

$$\frac{dx}{dt} = V_{el} \quad (11)$$

$$\frac{dV_{el}}{dt} = \frac{1}{M}[A(P - P_a) - F_{res}] \quad (12)$$

$$\frac{df}{dt} = \frac{B}{D}P^a + K_x x + K_v V_{el} \quad (13)$$

We solved such system of equations with ODE45. This MATLAB function uses a Dormand-Prince algorithm, also known as Runge-Kutta, of order 4.5. It is an algorithm similar to the Runge-Kutta of order 4; however, it uses variable step-size to increase speed.

The initial conditions of the problem are that the projectile leaves the position of rest and the remaining fraction of the ballistic length of the propellant is the unit (no propellant burned).

In a validation phase of computational implementation, we used the resistive force profile available in the PRODAS database as input parameter. Then, we started using Equation 9.

4.2 Adjustment of model parameters

We performed the adjustment of the model parameters with an approach of inverse problems. For that, we solved the direct problem, that is, obtaining the internal ballistics curves, for different sets of parameters tested. An objective function is used to assess the quality of each set of parameters; depending on whether a global or local search is being carried out, we estimate the best trial population set and create a new population (new generation), or estimate a new search direction.

The objective function, expressed by Equation 14, assesses the difference between the Pressure-Time curve obtained by each set of parameters and the reference curve, obtained from PRODAS.

$$E_{rro} = \frac{\sum_{i=1}^{k} E_i}{k} \quad (14)$$

where, k refers to the number of points and E_i is given by the equation:

$$E_i = \frac{(P_{pdi} - P_{PRODAS})^2}{P_{ref}} \quad (15)$$

The parameter estimation problem is solved by applying optimization methods to minimize a function that measures the distance of the system behavior in relation to a reference behavior, in the case of this study, Equation 14.

We chose to combine a global search method to find the region where the global minimum may be present with a local search method, so that they approach with a higher convergence rate.

For the global search, we used the MATLAB function that implements the genetic algorithms "*ga*" [8], while for the local search we used the "*fminunc*" function, which implements a quasi-Newton method based on the BFGS method [9], [10] and [11].

5. Results

We analyzed the case for the M80 7.62 × 51 mm ammunition in a barrel of 609.6 mm of ballistic length. Other parameters related to the propellant and the weapon that are not being adjustment objects – they are considered known – are presented in **Table 1**.

Tab. 1 – Weapon and propellant parameters used as input data

Input data		
Parameter	Symbol	Valor
Ballistic length	D	0.2667 mm
Form factor	k	0
Specific mass of the propellant	d_{ensi}	1578 kg/m³
Initial mass of the propellant	C	2.67 g
Adiabatic flame temperature	T_o	2825 K
Covolume of gases	c_{ovol}	0.001 m³/kg
Propellant force constant	F	0.9774 MJ/kg
Pressure index (exponent)	a	0.69
Ratio for specific heats	γ	1.24
Caliber	Ca	7.62 mm
Projectile mass	M	9.4876 g
Chamber volume	V_a	3.27761 cm³
Barrel length	L	609.6 mm
Rifling angle	α	4.49°
Weapon temperature	T_s	300 K
Minimum pressure to start the projectile	P_{min}	7.57 MPa
Moment of inertia	I_o	6.8861×10^{-8} kg·m²

Table 2 shows the parameters adjusted compared to the PRODAS reference curve. They allowed the objective function to return to 0.0262.

Tab. 2 – Parameters obtained by adjustment

Adjusted parameters			
B	K_x	K_y	μ
2.499e−7	4493.32	−1.95479	0.09955
$x_{minmesh}$	$x_{maxmesh}$	F_{mesh}	F_{obtu}
1.429e−05	3.603e−4	2324.97	162.43

The muzzle velocity, according to the adjusted model, was 897.53 m/s, representing a relative error in the order of 3.16% when compared with the reference value of 870 m/s presented by the model implemented by PRODAS. Understanding that the adjustment of parameters with objective function from Eq. 12 considers only the data regarding pressure, we considered the good agreement in the velocity profile, presented in **Figure 3**, a good indication of the quality of the model.

In **Figure 4**, we saw a good agreement between the adjusted pressure curve and the reference pressure curve. In view of the difference in the models, we expected some divergence in their overall behavior.

Figure 5 explains one of the main reasons for the divergence in the behavior of the pressure curve over time: the resistive force to the movement of the projectile according to its position in the bore has different characteristics from those present in the model used to generate the reference curve for the pressure-time curve.

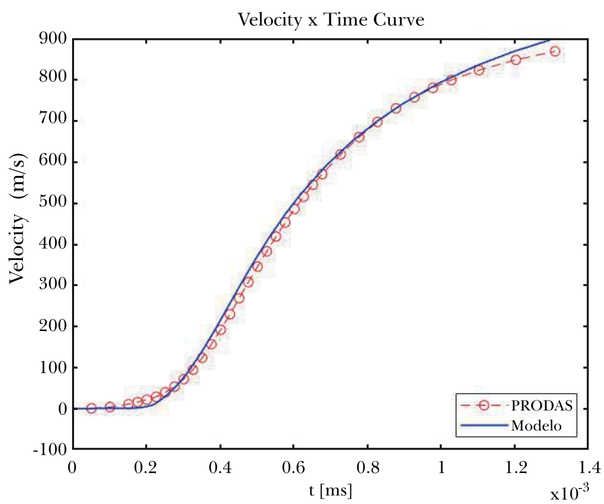

Fig. 3 – Velocity for time graph for 7.62 mm ammunition

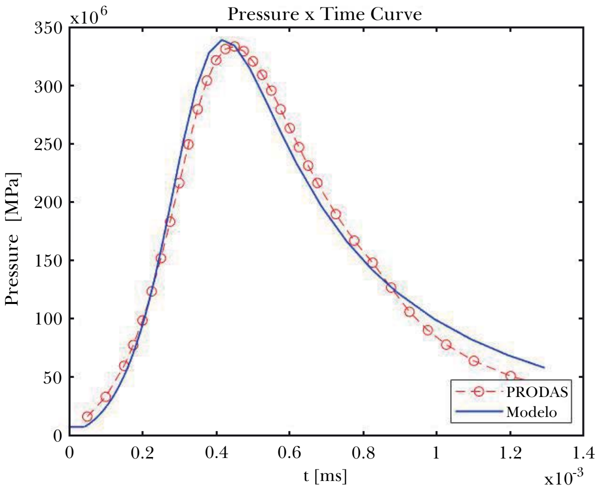

Fig. 4 – Pressure for time graph for 7.62 mm ammunition

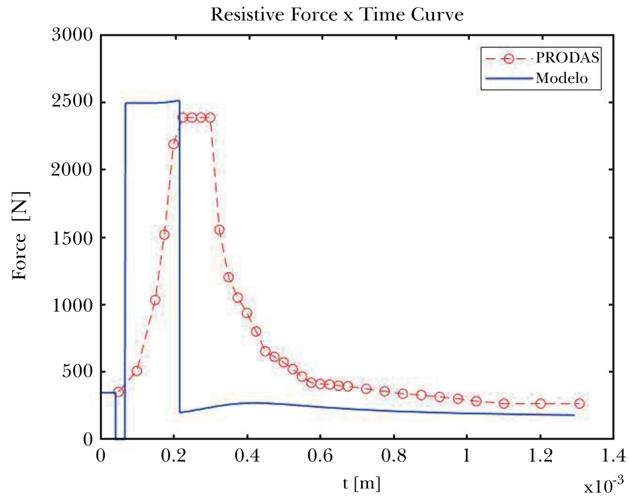

Fig. 6 – Resistive force for time graph for 7.62 mm ammunition

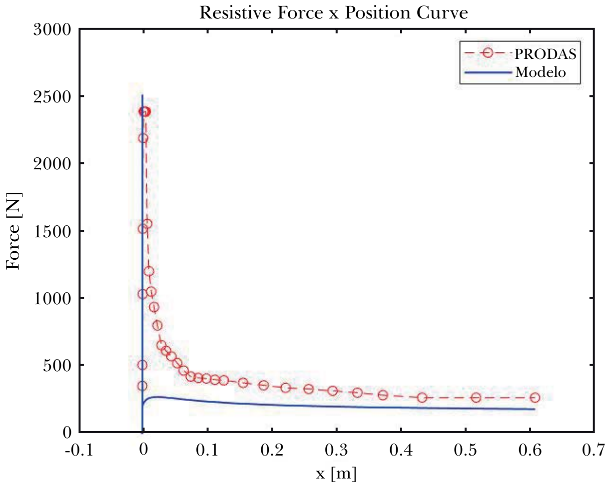

Fig. 5 – Resistive force for position graph for 7.62 mm ammunition

The resistive force curve as a function of time (**Figure 6**) allows a better visualization of its different components, when each of the forces becomes active. Initially, we see a constant force, referent to the force required to embed the projectile in the case; if there is a brief advance without resistance, until the projectile finds the forcing cone; the projectile is subjected to the mesh effort until the riflings are engraved on it and it reaches the same diameter as the bore. Along the bore, there is a constant component, due to the friction of the surface of the projectile with the bottom of the rifling, obturation; and a variable force, induced by the rifling, which is a function of the pressure of the gases and the acceleration of the projectile.

For executing the model in PRODAS, we need to have the resistive force profile according to the position of the projectile. It is an input parameter without direct correlation of it with physical parameters. In the proposed approach, the resistive force profile is function of physically significant parameters. However, due to the absence of data, they were obtained by adjustment of parameters to approach the pressure-time curve of the model to the curve resulting from the implementation of the Baer-Frankle model, implemented by PRODAS with the parameters available in its sample database.

6. Conclusions

We updated the modeling proposed by [4] to consider an expression for its resistive force based on the physical characteristics of its assembly in the cartridge, and the interaction of the projectile with the different parts of the bore.

The proposed procedure for adjusting the parameters based on the time evolution of the pressure was effective. We evidenced the quality of the model, and its numerical implementation, by the agreement of its result regarding the time evolution of the projectile velocity, an item that we did not consider in the adjustment methodology.

The modeling and the procedure presented proved to be promising, and is should be further explored by an analysis of its quality, strengthened by ballistic assays with different loadings and ammunition.

Acknowledgements

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