

# A statistical methodology for reducing wear and tear on railway tracks and comparing it with real wear and tear

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**ABSTRACT:** Railway tracks represent one of the most expensive assets for railways, and constant studies aim to extend their useful life. This work proposed equations for  $W1$ ,  $W2$ , and  $W3$  (variation of rail billet dimensions vertically, horizontally, and with an inclination of  $45^\circ$ ) to understand wear and tear. Using real data, the equations were developed and applied to an eight-year-old rail, revealing significant differences between measured and expected values. This discrepancy can be attributed to variations between railway sections and the statistical model used. It is suggested that a more detailed approach consider individual characteristics of each section and additional variables. This highlights the need for personalized analysis of track wear, considering the diversity of conditions along the railway.

**KEYWORDS:** Railway track. Useful life projection. Simple linear regression. Service life. Wear and tear positions.

**RESUMO:** Os trilhos ferroviários representam um dos ativos mais dispendiosos para as ferrovias, e constantes estudos visam prolongar sua vida útil. Este trabalho propôs equações para  $W1$ ,  $W2$  e  $W3$  (variação de dimensões do boleto do trilho na vertical, horizontal e com inclinação de  $45^\circ$ ) para entender o desgaste. Usando dados reais, as equações foram desenvolvidas e aplicadas a um trilho com oito anos de uso, revelando diferenças significativas entre os valores medidos e esperados. Essa discrepância pode ser atribuída às variações entre os trechos ferroviários e ao modelo estatístico utilizado. Sugere-se que uma abordagem mais detalhada considere características individuais de cada trecho e variáveis adicionais. Isso destaca a necessidade de uma análise personalizada do desgaste dos trilhos, levando em conta a diversidade das condições ao longo da ferrovia.

**PALAVRAS-CHAVE:** Trilho ferroviário. Projeção de vida útil. Regressão linear simples. Vida útil em serviço. Posições de desgaste.

## 1. Introduction

Rails represent one of the most valuable components in the Brazilian railway sector, mainly attributed to the nature of their material, the complex manufacturing process and the lack of national production. Consequently, extending the useful life of this asset is a priority for railway companies, aiming to minimize and postpone additional costs. To achieve this goal, preventive maintenance practices are employed to reduce wear and tear and prevent the appearance of Rolling Contact Fatigue (RCF).

Rail wear and tear is the reduction of the cross-sectional area over its period of use, which creates

a direct effect on the wheel-rail contact. This process results in the alteration of the mechanical properties of the material, which can adversely affect the dynamics of the railway vehicle, compromising reliability and safety of the permanent track [1-3].

Wear and tear of the railway is influenced by many factors, including:

1. Axle load: Studies have shown the relevance of axle load in the wear and tear rate of the rails [4; 5; 6];
2. Rail location: The position of the rail, such as in tunnels and curves, impacts its wear and tear [6];
3. Radius of curvature of the road: The radius of curvature directly influences the wear and tear of the rails [7];

4. Track geometry: The configuration of the railway track is a critical factor [8];
5. Speed of the compositions: The speed at which trains operate affects the wear and tear of the tracks [3];
6. Metallurgical rail properties: The characteristics of the rail material are decisive [6];
7. Dynamics of wheel-rail contact: The interaction between the wheel and the rail is crucial to understanding wear and tear [9];
8. Coefficient of friction: This factor is essential to understand the forces at stake [9];
9. Wheel and rail profile: The interaction of the rail and wheel profile directly influences wear and tear due to the contact ellipse [10].

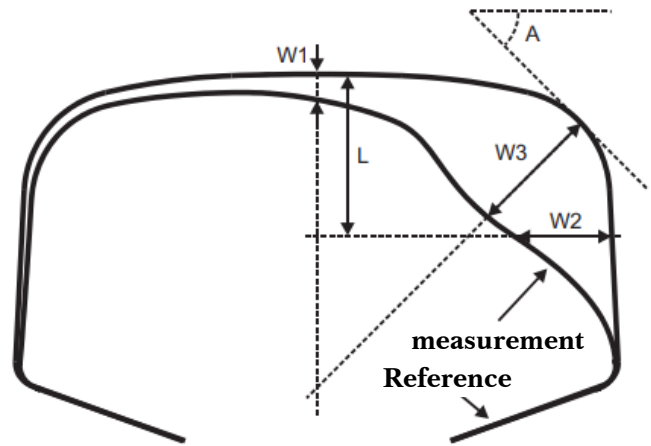
These studies highlight the complexity and multi-dimensionality of the phenomenon of wear and tear on railway tracks, underlining the importance of a holistic approach to the maintenance and layout of railways to minimize such effects.

Railways wear and tear is usually manifested in two main ways: vertical and horizontal/lateral. The vertical one is a widely observed phenomenon, which results from friction, abrasion, and the load imposed by the wheels of wagons and locomotives. This type of wear and tear leads to a progressive reduction in the height of the rail, potentially compromising the stability of the railway [11].

On the other hand, horizontal, or lateral, wear and tear affect the side of the rails, particularly at the top and base. This wear and tear is mainly induced by horizontal forces generated, e.g., when trains run on curves [11].

For the detailed analysis of wear and tear on rails, a study proposed a measurement method that identifies three critical areas, called **W1**, **W2**, and **W3**. **W1** corresponds to the change in the vertical profile at the top of the rail. **W2** refers to horizontal variation, whereas **W3** indicates the change measured at an angle of 45° [12]. Figure 1 shows the illustration of a rail, which clarifies the exact location of each measurement parameter, providing a clear view of the critical points for evaluation of wear and tear.

**Figure 1 -** Wear and tear analysis parameters.



**Source:** Adapted [13].

RCF defects arise from cyclic interactions between wheels of the train and rails during transport. These interactions result in shear forces that create surface defects in both the rails and wheels. The surface layer affected by these stresses undergoes a plastic deformation, resulting in an increase in hardness and changes in the microstructure of the material, due to hardening. This process can lead to the appearance of superficial cracks that can evolve and cause component failure, if not properly treated.

RCF is characterized by specific patterns of failures that manifest as nonconformities on the surface of the rails. Typical failures associated with contact fatigue include:

- Head checks: These are small cracks that appear at the corner of the rail gauge, arranged approximately 45° in relation to the rolling direction, resulting from cyclic stress on the surface [14];
- Shelling: As described by Rice [apud 15], this phenomenon occurs when head check type cracks deepen and expand inside the rail, reaching significant dimensions. This can lead to scaling of the rail material, which is a process known as shelling;
- Squats: They are the detachment of material from the surface of the rail, forming cavities. This type of defect is often associated with localized stress and accelerated wear and tear [14].

These failures highlight the importance of preventive maintenance practices and continuous monitoring of the railway infrastructure, aiming to mitigate the harmful effects of the RCF and ensure the safety and longevity of the railway system.

The adoption of strategies such as lubrication at the point of contact between wheel and rail and grinding represent effective methods in preventing and attenuating the effects of wear and tear and RCF [16].

The application of lubricants plays a crucial role in reducing and optimizing friction at wheel-rail contact, contributing significantly to decreasing wear and tear on these critical components. There are mainly two types of lubricants used for this purpose, namely:

- Friction modifiers: They are applied to the top of the rail to adjust the level of friction between the wheel and the rail, thus minimizing vertical wear and tear. The use of friction modifiers helps to balance the interaction between the wheels and the rails, extending their useful life [17];
- Grease: It is applied to the sides of rails, particularly in tight radius curves. The function is to mitigate horizontal or lateral wear and tear by protecting the rails against the abrasive effects of lateral forces exerted during the passage of trains [18].

Studies and practices in the field demonstrate the effectiveness of these approaches in maintaining the integrity of rail systems [19; 20]. The appropriate choice and careful application of lubricants are key to ensuring the efficiency of these preventive measures, standing out as vital components in the management of railway maintenance.

Rail grinding is a crucial preventive maintenance technique aimed at removing surface layers that have been deformed due to constant train traffic. This procedure is essential to restore the optimal geometry of the rail and improve the interaction between the wheel and the rail, eliminating defects such as RCF that can compromise the stability of the trains during circulation [21; 22].

Besides grinding and lubrication, designing and estimating the proper time for in-service rail replacement emerges as a significant proactive strategy. This approach aims to predict the remaining life of the

tracks, enabling planning in advance for the replacements needed to maintain the safety and operational efficiency of the track. Such an estimate is critical for rail asset management, ensuring that operations are conducted safely and cost-effectively, minimizing disruptions and maximizing the longevity of the infrastructure.

This study aimed to compare the durability of the TR-68 rail in terms of wear and tear, using both projections based on statistical methods—specifically Pearson’s linear correlation and simple linear regression—based on hard data, and the analysis of the cross-section of a rail withdrawn from operation after eight years due to a fracture. The results revealed that it is not feasible to define a single equation for each wear and tear position, capable of determining the useful life of the rail along the entire length of a railroad. This is due to significant variables, such as the characteristics of the terrain, the volume and type of cargo transported—and the speed of the railway compositions that are specific to each stretch and significantly impact the forecast of the asset’s durability. This study also suggests that the complex reality of rails requires the consideration of a broader set of variables, indicating that simple linear regression may not be the most accurate methodology for this purpose.

## 2. Statistical methodology for predicting the useful life of rails

Initially, this study focused on the analysis of the correlation level between rail wear and tear and the volume of cargo transported, expressed in Millions of Gross Tons Transported (MGTT). To this end, Equation 1 determined the Pearson’s correlation coefficient ( $r$ ) used, aiming to quantify the degree of association between these two variables.

$$r = \frac{n \cdot \sum x_i y_i - (\sum x_i) \cdot (\sum y_i)}{\sqrt{[n(\sum x_i^2) - (\sum x_i)^2] \cdot [n(\sum y_i^2) - (\sum y_i)^2]}} \quad (1)$$

In this context:

$n$  represents the total number of data pairs analyzed;

$x_i$  and  $y_i$  correspond, respectively, to the variables under study.

The aim of examining Pearson's correlation coefficient ( $r$ ) is to determine the level of linear relationship between the two variables in question,  $\mathbf{x}_i$  and  $\mathbf{y}_i$  [23].

Table 1 shows in detail several inferences that can be drawn by interpreting the value of  $r$ , which was calculated by Equation 1.

**Table 1** - Interpretation of the value obtained for Pearson's coefficient ( $r$ ).

Coefficient of Pearson's correlation ( $r$ )	Linear correlation
$r = 1$	Perfect positive
$0.8 \leq r < 1$	Strong positive
$0.5 \leq r < 0.8$	Moderate positive
$0.1 \leq r < 0.5$	Moderate positive
$0 \leq r < 0.1$	Negligible positive
0	Zero
$-0.1 \leq r < 0$	Negligible negative
$-0.5 \leq r < -0.1$	Weak negative
$-0.8 \leq r < -0.5$	Moderate negative
$-1 \leq r < -0.8$	Strong negative
$r = -1$	Perfect negative

**Source:** adapted [14].

When Pearson's correlation coefficient ( $r$ ) is equal to 0, there is an absence of linear correlation between the variables analyzed. On the other hand,  $r$  values close to 1 or -1 suggest a strong linear correlation between the variables, whether positive or negative, respectively. A positive correlation implies that as one variable increases, the other also increases, whereas a negative correlation indicates that the increase in one variable results in the decrease in the other. An  $r$  value equal to 1 or -1 denotes a perfect linear correlation, whether fully direct or inverse.

In addition to numerical analysis, the relationship between variables can be visualized graphically by means of a scatter plot. An arrangement of the variables that closely aligns with a straight line on the graph

indicates a strong linear correlation (with Pearson's coefficient close to 1). In contrast, a random distribution of the variables in the graph signals a weak linear correlation (Pearson's coefficient close to 0) [23].

Based on Pearson's coefficient and the observation of the scatter plot, the simple linear regression model is employed to elucidate the relationship between the two variables in focus. This model was also used to make projections about the useful life of the rails, seeking to provide a simple tool for future forecasts based on historical data.

Simple linear regression is a statistical method that aims to establish a mathematical equation to describe the relationship between two specific variables, which are classified as [25; 26]:

- Dependent variable ( $\mathbf{Y}$ ), also known as the response variable, of which the behavior is to be predicted or better understood;
- Independent variable ( $\mathbf{X}$ ), also called the explanatory variable, which is used to make predictions or explain the variations in the dependent variable.

Equation 2 outlines the theoretical model to construct this simple linear equation [27]. This model provides the basis for calculating the linear relationship between variables, enabling educated predictions and analysis on how one variable may affect the other.

$$Y = \alpha X + \beta + \epsilon_i \quad (2)$$

In which:

- $\mathbf{X}, \mathbf{Y}$ : variables under analysis;
- $\alpha$  and  $\beta$ : unknown parameters;
- $\epsilon_i$ : random error.

The values of  $\alpha$  and  $\beta$  can be obtained by Equations 3 and 4 [27].

$$\beta = \frac{n \cdot \sum x_i \cdot y_i - (\sum x_i) \cdot (\sum y_i)}{n \cdot (\sum x_i^2) - (\sum y_i^2)} \quad (3)$$

$$\alpha = \bar{y} - \beta \cdot \bar{x} \quad (4)$$

In which,  $\bar{x}$  is the mean of the values of  $\mathbf{x}_i$  and  $\bar{y}$  is the mean of the values of  $\mathbf{y}_i$ .

### 3. Data collection

The data used in this statistical analysis, which covered linear correlation and simple linear regression,

were collected between 2019 and 2022. The research focused on the Vignole TR-68 profile rail, using specific wear and tear measurements (**W1**, **W2** and **W3**). Figure 2 shows that the MiniProf equipment was used to measure wear and tear.

**Figure 2** - Field measurement with the mini prof.



**Source:** Author.

The data compiled for this research reflect specific characteristics of the railway system studied, including:

- Estimated annual MTBT: ~ 150;
- Axle for transported load: 32 tons;
- Flow: Heavy Haul loaded;
- Installation position: External rail in medium radius curve.

In addition, the investigation included the analysis of a rail sample that was replaced due to a fracture after eight years of continuous use. Figure 3 illustrates the cross-section of the affected rail, showing initial signs of cracking by shelling and subsequent progression due to fatigue.

**Figure 3** - Cross-section of the rail sample used in this research.



**Source:** Author.

The following data are from this sample:

- Estimated annual MTBT: ~ 50;
- Axle for transported load: 32 tons;
- Flow: Heavy Haul loaded;
- Installation position: Tight radius curved outer rail.

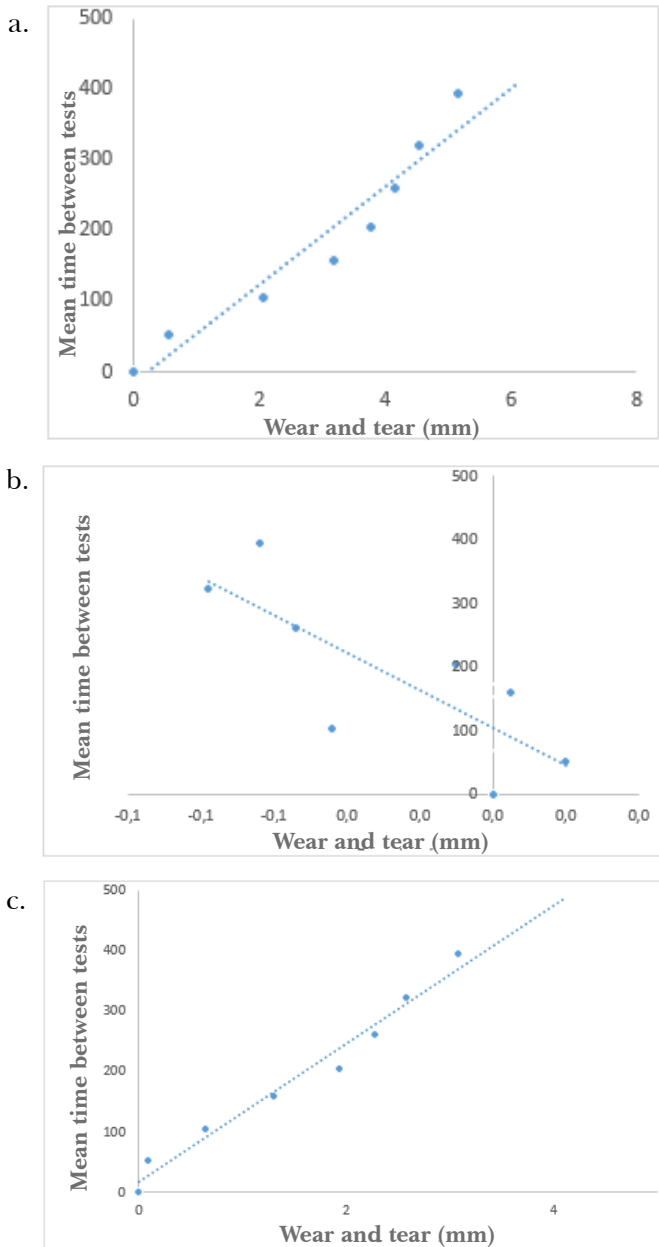
## 4. Results and discussion

Figure 4 illustrates the scatter plots and trend lines that were plotted using Excel to examine possible visual relationships between MTBT and wear and tear on railways, considering each assessed wear and tear position.

Note that, in the **W1** and **W3** (Figures 4a and 4c) positions, there is a linear relationship between the two variables. In contrast, for the position **W2** (Figure 4b), according to the data collected in the field, the observed wear and tear was almost non-existent, resulting in negative values in the MiniProf. Studies have shown that the application of the third body in the wheel rail contact can eliminate the lateral wear generated in curves with radius between 437 and 159 m [16].

Table 2 shows the degree of association between the variables calculated, which determined the value of Pearson's correlation coefficient ( $r$ ), the coefficient of determination ( $r^2$ ), and the adjusted coefficient of determination.

**Figure 4** - Scatter plots with trend lines for positions (a) w1, (b) w2 and (c) w3.



Source: Author.

When analyzing Pearson's correlation coefficients ( $r$ ), determination ( $r^2$ ), and the adjusted determination shown in Table 2, note that the values for **W1** and **W3** are very close to 1, suggesting a strong and positive linear correlation between the variables (Table 1). On the other hand, for **W2**, a positive linear correlation of moderate intensity is observed (Table 1), which

is in line with the pattern in the scatter plot for this position (Figure 4b). This means that for **W2** position, approximately 79% of the variation in the dependent variable can be explained by the independent variables included in the model. To the positions **W1** and **W3**, this explainability is higher, exceeding 90%.

**Table 2** - Values of the Person coefficients ( $r$ ), determination ( $r^2$ ) and adjusted determination for the three wear and tear positions.

Position	$r^2$	$r$	Adjusted $r^2$
<b>W1</b>	0.9287035	0.9636926	0.916820753
<b>W2</b>	0.624447	0.7902196	0.561854795
<b>W3</b>	0.9746352	0.9872361	0.970407679

Source: Author.

Table 3 described the estimated values for the parameters  $\alpha$  and  $\lambda$  obtained with the support of Excel, derived from the linear regression model, as well as the associated standard error.

**Table 3** - Linear regression coefficients.

	Model	Coefficients	Standard error
<b>W1</b>	<b>MTBT</b>	15.485	26.736
	<b>Wear and tear</b>	69.279	7.8365
<b>W2</b>	<b>MTBT</b>	104.32	41.068
	<b>Wear and tear</b>	2941.5	931.29
<b>W3</b>	<b>MTBT</b>	16.545	41.068
	<b>Wear and tear</b>	114.675	931.29

Source: Author.

The equations shown in Table 3 associated with the adjusted regression model, obtained with the support of Excel, are as follows:

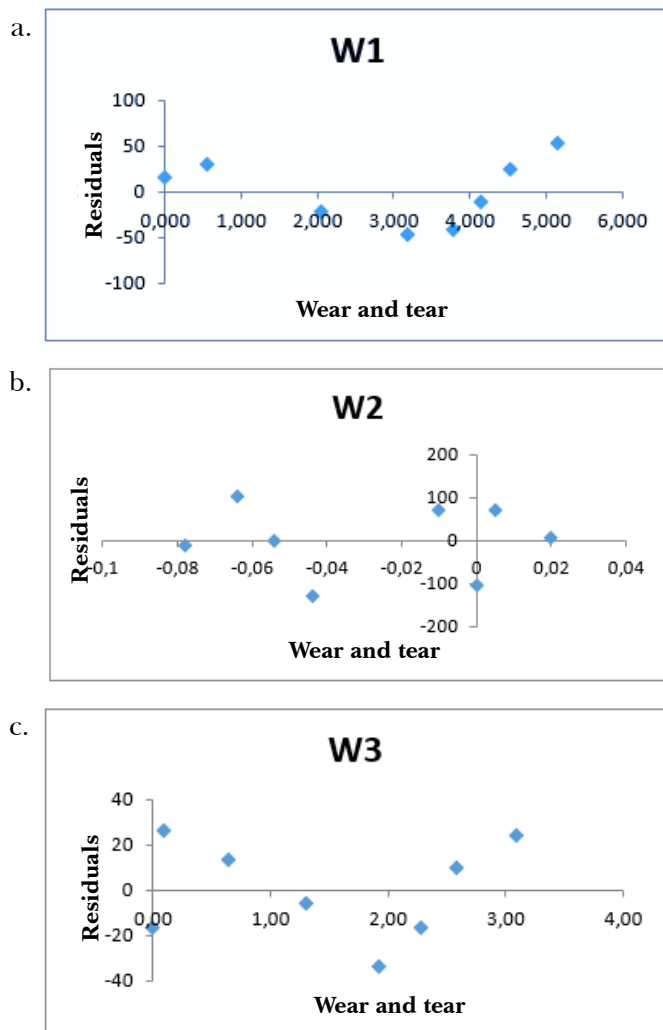
$$W1, MTBT = 69.279Wear\ and\ tear - 15.485 \quad (5)$$

$$W2, MTBT = -2941.5Wear\ and\ tear + 104.32 \quad (6)$$

$$W3, MTBT = 114.67Wear\ and\ tear + 16.545 \quad (7)$$

Figure 5 shows graphs of residuals in relation to rail wear and tear. Residuals are the differences between the observed values and the values predicted by the regression model. Analysis of these results can provide valuable insights into the model's suitability for the actual data [28].

**Figure 5** - Graphs of residuals of the three wear and tear positions under analysis, in which (a) refers to W1, (b) refers to W2, and (c) refers to W3.



Source: Author.

Analyzing the graphs of the residuals, it is verified that they have an asymmetric distribution. The graphs referring to the **W1** and **W3** positions (Figures 5a and 5c, respectively) show a positive asymmetry, whereas the graph for the **W2** position (Figure 5b) has a negative asymmetry.

Asymmetry in the residuals may indicate that the linear regression model is not ideal to describe the relationship between the variables or that there is no linearity or heteroscedasticity not captured by the model. This may point to the need for adjustments, such as the inclusion of nonlinear terms, variable transformations, or consideration of other forms of modeling [28].

Table 4 shows the ANOVA results (p-value) for each of the positions that were verified after the analysis of the residuals.

**Table 4** - ANOVA results.

Variable	p-value
<b>W1</b>	0.0001
<b>W2</b>	0.0196
<b>W3</b>	5.15E-06

Source: Author.

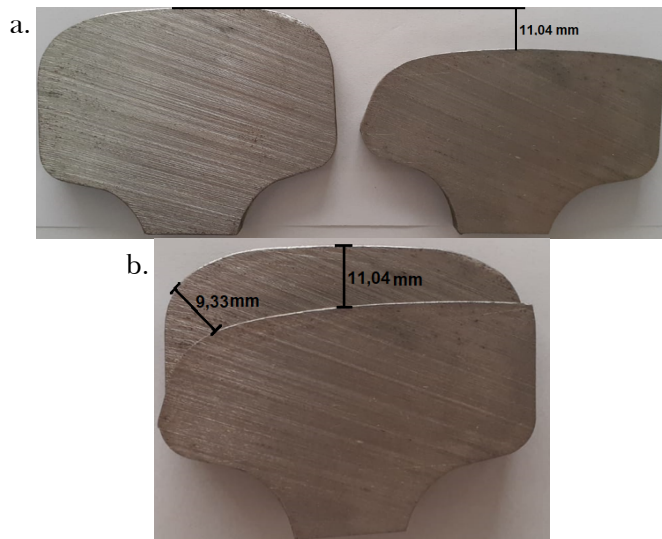
For p-values above 0.050, we do not have evidence to support the assertion that the linear regression model is significant to explain the variable of interest, indicating that independent variables do not influence the dependent variable. However, if the p-value is  $\leq 0.050$ , we can infer that at least one of the variables is relevant to explain the dependent variable, i.e., there is an influence of at least one independent variable on the dependent variable [28]. So, when examining Table 4, all p-values are less than 0.05. Therefore, we conclude that regression is statistically significant, indicating the importance of the linear regression model to explain the dependent variable.

Using Equations 5 and 7 and excluding the data referring to the **W2** position due to the low wear rate observed during the analysis period—as evidenced in Figure 4b—the estimated calculation for the wear and tear profile of the railway that was in operation

for eight years was performed (Figure 3). The results showed an estimated wear and tear of 5.82 mm in the **W1** position and 3.24 mm in the **W3** position.

Figure 6 shows that the actual wear and tear experienced by this rail was measured by a caliper and a protractor to obtain the values of **W1** and **W3**.

**Figure 6** - (a) Profile of the new and used rail side by side and (b) profile of the new and used rail superimposed with the appropriate measurements of **W1** and **W3**.



**Source:** Author.

When evaluating the calculated values for **W1** (5.82 mm) and **W3** (3.24 mm) using Equations 5 and 7 and comparing them with the actual values recorded for the rail being used (**W1** = 11.04 mm and **W3** = 9.33 mm), there is a considerable discrepancy. This disparity can be attributed to the specific location of each rail on the railway. The section used to derive the equation has a curve with a mean radius of more than 900 m and approximately 12 MTBT per month. In contrast, the sample that provided the actual values of **W1** and **W3** (Figure 3) was installed in a section with a tighter radius curve, less than 250 m and approximately 4 MTBT per month. In addition to the divergences in the geometric design of the road, there is a significant disparity in the volume of cargo transported.

However, despite the discrepancy in the absolute values found, the relative values remained consistent with the actual data, i.e., **W1** > **W3**.

Thus, note that, in this case, a single equation based on simple linear regression for each position of analysis (**W1**, **W2**, and **W3**) is inadequate to describe the behavior of the entire railroad under study. The most appropriate would be to obtain equations using the multivariable regression methodology that would consider several parameters, such as radius of curvature, MTBT, ramp, and lubrication. Additionally, due to the particularities in each section of the railway, it would be advisable to show specific equations for each region, considering their individual characteristics.

It is relevant to highlight that methodologies related to modifications in wheel-rail contact, such as changes in wheel and rail profiles, the application of friction modifiers, and the use of grease were not included in the statistical calculations.

## 5. Conclusion

- The measured values of wear and tear of the rail that showed a fracture were higher than those calculated for the same rail using the equations developed in this study. However, the correlation indicating that **W1** is greater than **W3** was consistently observed;
- The simple linear regression approach adopted failed to fully capture all factors in the rail infrastructure that affect rail wear and tear. Therefore, there is a need to refine the model by including other variables that are significant to explain the observed phenomenon;
- Wear and tear is intrinsically linked to the conditions of the environment in which the asset is inserted. Therefore, the discrepancy between the values measured on the fractured rail and those obtained by the equation can be attributed, in part, to the location of the rails. The fractured rail was in an environment with conditions completely different from those of the rail used as a reference for the formulation of the equations. This may indicate that the same equation, for each of the wear and tear positions (**W1**, **W2**, and **W3**), is not adequate to describe an entire railway in detail.

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